CALCULATION OF THE MOTION OF RELATIVISTIC BEAMS OF CHARGED PARTICLES IN ELECTROMAGNETIC FIELDS

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In calculating high-current relativistic beams of charged particles moving in electromagnetic fields, it is necessary to take account of the effect of the electric and magnetic self-fields products by the beams themselves. This effect has been modeled on a computer [1, 2]. The present paper describes numerical algorithms contained in the KSI-BÉSM compiling system [3] which permit the inclusion of a broad class of relativistic problems, taking account of the magnetic field of currents flowing in the metal parts of the device being calculated, and also problems with virtual cathodes.

The axisymmetric problems considered are formulated as follows. The potential φ of the electric field satisfies Poisson's equation

$$\Delta\varphi = -4\pi\rho \tag{1}$$

in the closed domain $\bar{G} = G + \Gamma$, where ρ is the space charge density, and $\Delta = \frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial}{\partial x}\right) + \frac{\partial^2}{\partial y^2}$ is the Laplace operator. Boundary conditions of the first or second kind are specified for the potential on portions of the boundary of Γ. Internal boundaries between two media with different dielectric constants may occur in the computational domain G. The equation of motion of a particle of rest mass m and charge e is

$$\frac{d}{dt}\left(\gamma m\frac{d\mathbf{r}}{dt}\right) = e\mathbf{E} + \frac{e}{c}[\mathbf{v}\times\mathbf{H}], \tag{2}$$

where E and H are the electric and magnetic field intensities, $\mathbf{r} = (x, y)$ is the radius vector, $\mathbf{v}$ is the velocity of the particle, c is the velocity of light, $\gamma = (1 - v^2/c^2)^{1/2}$ is the relativistic factor, and $v = |\mathbf{v}|$. It is assumed that $\mathbf{H} = \mathbf{H}_e + \mathbf{H}_s$, where $\mathbf{H}_e$ is the external magnetic field and $\mathbf{H}_s$ is the magnetic self-field produced by the beam and currents in metal. The current density $\mathbf{j}$ of the particle beam under study at the entrance boundary of the computational domain is either specified by a function of coordinates or is determined in the process of solving the problem by the "3/2" law [4], assuming that the condition

$$\operatorname{div}\mathbf{j} = 0 \tag{3}$$

is satisfied over the whole domain.

We circumscribe a rectangle about the computational domain G and construct within it a net Ω_h formed by the lines $x = x_i$ and $y = y_j$. Let $h_i^x = x_{i+1} - x_i$, and $h_j^y = y_{j+1} - y_j$ be the steps of the net, and $h = \sup_{x,y}\max_{i,j}(h_i^x, h_j^y)$. Equation (1) and the boundary conditions on the potential φ are approximated on Ω_h by a system of difference equations [4]. For example, for internal nodes $(i, j) \in \Omega_h$ which are not close to the boundaries the difference equations have the form

$$\frac{2x_{i-\frac{1}{2}}}{x_i h_{i-1}^x\left(h_{i-1}^x + h_i^x\right)}\varphi_{i-1,j} + \frac{2x_{i+\frac{1}{2}}\varphi_{i+1,j}}{x_i h_i^x\left(h_{i-1}^x + h_i^x\right)} + \frac{2\varphi_{i,j-1}}{h_{j-1}^y\left(h_{j-1}^y + h_j^y\right)} + \frac{2\varphi_{i,j+1}}{h_j^y\left(h_{j-1}^y + h_j^y\right)} - 2\left(\frac{1}{h_{i-1}^x h_i^x} + \frac{1}{h_{j-1}^y h_j^y}\right)\varphi_{i,j} = 4\pi\rho_{i,j},$$

where $\varphi_{i,j}$ are the approximate values of the potential at the nodes of Ω_h. The system of difference equations is solved by iteration methods which involve a succession of finer and finer mesh nets [5-7]. In order to solve the problem on the net Ω_h, an auxiliary problem is first solved on a coarser net Ω_{h_1} $(h_1 > h)$ with a smaller

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number of nodes. The solution obtained is interpolated at the nodes of Ω_h and used as the initial approximation for the iterative process for finding the final solution on the net Ω_h. Theoretical estimates and the solution of practical problems show that the solution can be obtained to a given accuracy by this approach with many fewer arithmetical operations than are required in a calculation on a fixed net.

The equations of motion (2) are integrated numerically by the following difference scheme:

$$\mathbf{v}_{p+1} = \Delta t_p \frac{e}{m} \sqrt{1-\beta_{p+\frac{1}{2}}} \left(\mathbf{E}_{p+\frac{1}{2}} + \left[\frac{\mathbf{v}_p + \mathbf{v}_{p+1}}{2c} \times \mathbf{H}_{p+\frac{1}{2}} \right] - \frac{1}{2c^2} \times \left((\mathbf{v}_{p+1} + \mathbf{v}_p) \cdot \mathbf{E}_{p+\frac{1}{2}} \right) \frac{\mathbf{v}_{p+1} + \mathbf{v}_p}{2} \right) + \mathbf{v}_p, \quad \mathbf{r}_{p+1} = \mathbf{r}_p + \Delta t_p \frac{\mathbf{v}_{p+1} + \mathbf{v}_p}{2}, \tag{4}$$

where Δt_p is the step of the numerical integration, and the subscript $p+1/2$ indicates that the quantity is evaluated at the midpoint $\mathbf{r}_{p+1/2} = \mathbf{r}_p + (\Delta t_p/2)\mathbf{v}_p$, $\beta = v^2/c^2$, with v given by $v_{p+1/2} = \left(1 - \left(\frac{|e|\varphi_{p+1/2}}{c^2} + 1\right)^{-1}\right)^{1/2}$, $\varphi \geqslant 0$. This difference scheme has an error $0(\Delta t^2)$, where $\Delta t = \max\limits_p \Delta t_p$.

The space charge is calculated by the "current tube" method [8, 9], which is an economical variant of the widely known method of "large" particles.

In order to solve the so-called self-consistent problem defined by Eqs. (1)-(3) with initial and boundary conditions, an iterative process with respect to space charge is constructed. A process of the following form is commonly used:

$$\Delta\varphi^{n+1} = -4\pi\rho^n, \quad \rho^n = \omega\rho^{n,1} + (1-\omega)\rho^{n-1}, \quad n = 1, 2, \ldots, \tag{5}$$

where $\rho^{n,1}$ is the value of the space charge obtained by calculating the trajectories in the field with the potential φ^n; $0 < \omega \leq 1$ is the relaxation parameter of the successive approximations. Another possible algorithm of the process of successive approximations with respect to space charge is given in [10].

We consider in more detail the algorithm for calculating the magnetic self-field of a beam of charged particles. In the equations of motion we take account only of the azimuthal component of the magnetic self-field of the beam

$$H_\psi = 2I/cR,$$

where I is the current through a cross section of radius R, assuming the other components are negligibly small. We note that in the absence of an external magnetic field the self-field of the beam contains only the azimuthal component, so that for this case the magnetic field of the beam is taken into account completely in the algorithm.

The magnetic field of the beam is calculated by separating out of domain G a subdomain G' into which the beam under study penetrates. In subdomain G' a rectangular nonuniform net Ω'_h is constructed with $\Omega'_h = \{x = x'_i,\ y = y'_j,\ i = 0, 1, \ldots, N_1,\ j = 0, 1, \ldots, N_2\}$ with steps $\hbar^x_i = x'_{i+1} - x'_i$, $\hbar^y_j = y'_{j+1} - y'_j$. We divide the initial beam front, i.e., the boundary where the particles enter the computational domain, into N_T current tubes (trajectories), each of which carries a current I_k ($k = 1, 2, \ldots, N_T$). Suppose for definiteness that the beam travels in the direction of the y axis and $\mathbf{r}^k_p$ and $\mathbf{r}^k_{p+1}$ are calculated points on the k-th trajectory obtained in the integration of the equations of motion by scheme (4), where $\mathbf{r}^k_p$ and $\mathbf{r}^k_{p+1} \in G$. If a line $y = y'_j$ of the net Ω'_h is found such that $y^k_p \leq y'_j \leq y^k_{p+1}$ (or $y^k_{p+1} \leq y'_j \leq y^k_p$) the current of the trajectory I_k (or $-I_k$) contributes at the (i, j)-th node closest to the point $\mathbf{r}^k_{p+1/2} = \frac{\mathbf{r}^k_{p+1} + \mathbf{r}^k_p}{2}$. It is important to choose the integration step small enough so that the trajectories do not intersect several lines of the net during one step. If a line of the net is not intersected, the current of the trajectory does not contribute.

At the first step of the integration ($\mathbf{r}^k_p$ is the initial point) the current of the trajectory is added to all the nodes of the net which have coordinates $y'_j \leq y^k_p$ and lie on the line $x = x'_i$ closest to the point $\mathbf{r}^k_p$.

If a trajectory is incident on any metal surface, it is determined whether this is a surface of the cathode part of the device. If this is the case, i.e., if $\mathbf{r}^k_{p+1}$ is an external point belonging to the cathode region, the current of the trajectory I_k is subtracted from the nodes (x'_i, y'_j) ($i = \text{const}$, $j \leq j_1$), where (x'_i, y'_j) is the node closest to the point $\mathbf{r}^k_p$. If the external point $\mathbf{r}^k_{p+1}$ belongs to the anode region, the current of the trajectory is added to all the nodes (x'_i, y'_j) ($i = \text{const}$, $j \geq j_1$). Thus, after calculating all the trajectories at the nodes of the net (i,

TABLE 1

ν	3	3,5	4	4,5	5
Δr_A	0,0190	0,0258	0,0337	0,0427	0,0527
Δr	0,0173	0,0236	0,0312	0,0394	0,0476
δ, %	8,9	8,5	7,4	7,7	9,6

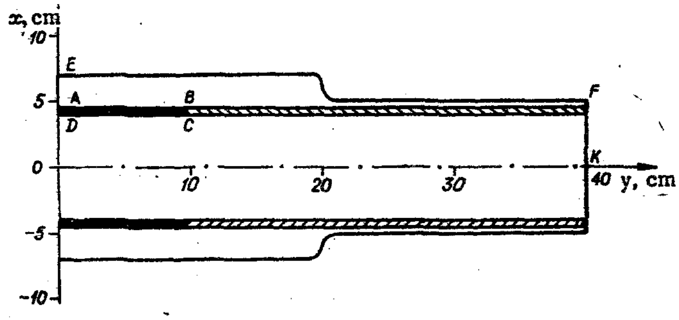

Fig. 1

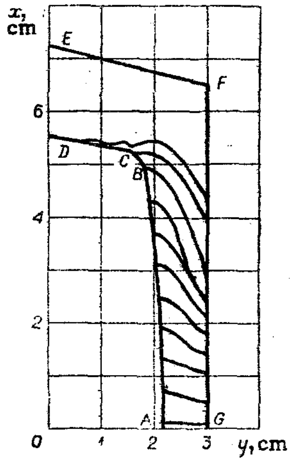

Fig. 2

j) $\in \Omega'_h$, a complete spatial pattern is obtained for the distribution of currents I_{ij} of the system under study.

The values of the magnetic self-field of the currents at points x'_i, y'_j are calculated from the expression

$$(H_\psi)_{ij} = \frac{2\sum\limits_{s=0}^{i} I_{sj}}{cx'_i}.$$

In solving the self-consistent problem in parallel with the process of successive approximations with respect to space charge (5), a relaxation of the magnetic field

$$H_\psi^n = \omega H_\psi^{n,1} + (1-\omega)\,H_\psi^{n-1}$$

is carried out, where $H_\psi^n = \{(H_\psi^n)_{ij}\}$ is the vector whose components give the values of the magnetic field at the nodes of Ω'_h, ω is the same quantity as in (5), and $H_\psi^{n,1}$ is the magnetic field produced by the n-th approximation currents. It is assumed that $H_\psi^0 = 0$. The trajectories of the n-th approximation are calculated in the field H_ψ^{n-1}, where the value of $(H_\psi)_{p+1/2}$ in Eq. (4) is found by linear interpolation of the values of $(H_\psi^{n-1})_{ij}$ at the nearest nodes:

$$(H_{\psi}^{n-1})_{p+\frac{1}{2}} = (H_{\psi}^{n-1})_{ij} + \frac{y_{p+\frac{1}{2}} - y'_j}{h_j^y} [(H_{\psi}^{n-1})_{ij+1} - (H_{\psi}^{n-1})_{ij}],$$

where (i, j) is the node determined by the inequalities $0 \leq x_{p+1/2} - x'_i \leq h_i^x$, $0 \leq y_{p+1/2} - y'_j \leq h_j^y$.

The algorithms considered are contained in the KSI-BÉSM library of compiling systems [3]. The present system permits the calculation of steady and unsteady [11] beams of charged particles of various signs and masses, taking account of the initial energy and angular distributions, the phenomena of secondary emission [12], and the electromagnetic self-field of the beam.

A number of problems were solved using the KSI-BÉSM system. The error of the algorithms described above was investigated by calculating the spreading of a beam of relativistic particles in an equipotential space. The following model was taken for the numerical calculation. A monochromatic electron beam with a current $I = 1$ kA having an initial radius $R_0 = 0.05$ cm and a velocity $v_0 = c\gamma^{-1}(\gamma^2-1)^{1/2}$ ($\gamma = 4$) enters a cylinder of radius $R = 1.5$ cm and length $l = 6$ cm. The condition $\partial\varphi/\partial \mathbf{n} = 0$ for the potential is specified at the surface where the particles enter and on the axis of the cylinder. At the other end of the cylinder and on its lateral surface $\varphi =$ 1500 kV. The beam moves along the axis of the cylinder which coincides with the y axis of the coordinate system. The net Ω_h is uniform along the y axis with $m_y = 120$ nodes. Along the x axis there are two zones with boundaries $x_0 = 0$, $x_1 = 0.7$ cm, $x_2 = 1.5$ cm containing $l_1^x = 28$ and $l_2^x = 10$ nodes. The net Ω'_h has the following parameters: $y_0 = 0$, $y_1 = 6$ cm, $m_y = 120$, $x_0 = 0$, $x_1 = 0.45$ cm, $x_2 = 0.8$ cm, $l_1^x = 9$, $l_2^x = 14$. The beam was modeled by 60 trajectories. Table 1 compares the values of the increments in the radius of the beam Δr obtained by numerical calculations with the approximate analytical values Δr_A [13] given by

$$\Delta r_A = \frac{3 \cdot 10^9 e I y^2}{m(\gamma\beta c)^3 R_0}$$

(I is the current in amperes, and all the remaining quantities are in cgs units) for certain values of the y coordinate.

It is clear from Table 1 that the relative error of the spreading of the beam $\delta = (|\Delta r - \Delta r_A|)/\Delta r_A$ is no more than 10%. The relative error of the calculation of the radius does not exceed 1%. This accuracy is adequate for most practical problems.

A calculation was performed with Belomyttsev [14] for a beam of particles in the diode space (Fig. 1) emitted from the surface ABCD with a space charge limited current density given by the "3/2" law. The boundary conditions are: $\varphi = 0$ on ABCD, $\partial\varphi/\partial \mathbf{n} = 0$ on AE, OD, FK, and OK (the axis of symmetry of the system), and $\varphi = \varphi_0$ on EF. It was required to find the minimum value of the external magnetic field H_z for which the beam electrons do not strike the walls of the drift chamber EF. For $\varphi_0 = 2.5$ MeV and $H_z = 10$ kG a beam having an initial width of 0.5 cm is broadened to 0.8 cm, so that the gap between the beam and the chamber wall is 0.2 cm. At the system exit the electrons of the inner layer have an energy of 2 MeV and those of the outer layer 2.3 MeV; i.e., the energy spread of the beam electrons is 15%. The maximum velocities of the electrons in this case reach c/3.

In calculating strong relativistic beams in the absence of external magnetic fields, the azimuthal magnetic field of the beam and the currents in metal exerts an appreciable effect on the behavior of the particles. This situation occurs, e.g., in problems of calculating magnetic insulation in the transmission of strong pulsed currents in vacuum coaxial lines [15], and also in focusing a high-current electron beam in a diode.

A diode node of the "Akvagen" accelerator [16] was calculated (Fig. 2). The cathode surface is a segment of a sphere AB joined to a conical surface CD. The radius of curvature of the connecting portion BC is 0.4 cm. The calculation was performed for zero potential of the cathode ABCD and a potential of the anode EFG equal to 1 MeV.

The calculation of the trajectories shown in Fig. 2 indicates an appreciable focusing of electrons under the influence of the azimuthal magnetic self-field of the beam. The angular spread of the electrons at exit is 60°. The calculated value of the total diode current is 100 kA, which is in good agreement with the value 130 kA estimated theoretically by the relativistic analog of the "3/2" law [17].

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